M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-I

PSCMTH03 : Topology-I

P. Pages : 2 | **GUG/W/22/13739**

Time : Three Hours | Max. Marks : 100

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Notes : 1. All **five** questions are compulsory.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that the union of a denumerable number of denumerable sets is a denumerable set. **10**

b) Prove that the set of all real numbers is uncountable. **10**

OR

c) Prove that $2^a > a$ for every cardinal number a. **10**

d) Prove that the set of all rational numbers is denumerable. **10**

UNIT – II

2. a) Prove that for any set E in a topological space, $c(E) = E \cup d(E)$. **10**

b) Prove that a set F is a closed subset of a topological space iff F^C is an open subset of the space. **10**

OR

c) Prove that for any set E in a topological space (X, τ), $i(E) = \left[C\left(E^C\right)\right]^C$. **10**

d) Prove that τ^* is a topology for X^*. **10**

UNIT – III

3. a) If C is a connected subset of a topological space (X, τ) which has a separation X = A|B, then prove that either $C \subseteq A$ or $C \subseteq B$. **10**

b) Prove that a compact subset of a topological space is countably compact. **10**

OR

c) If f is a continuous mapping of (X, τ) into $\left(X^*, \tau^*\right)$, then prove that f maps every compact subset of X onto a compact subset of X^*. **10**

d) If f is a homeomorphism of X onto X^*, then prove that f maps every isolated subset of X onto an isolated subset of X^*. **10**

UNIT – IV

4. a) Prove that a topological space X is a T_0 - space iff the closures of distinct points are distinct. **10**

b) Prove that every compact subset E of a Hausdorff space X is closed. **10**

OR

c) Prove that a topological space X satisfying the first axiom of countability is a Hausdorff space iff every convergent sequence has a unique limit. **10**

d) Prove that a topological space X is regular iff for every point $x \in X$ and open set G containing x there exists an open set G^* such that $x \in G^*$ and $C\left(G^*\right) \subseteq G$. **10**

5. a) Define **5**

i) Equipotent sets
ii) Denumerable set
iii) Countable set

b) Define **5**

i) Closed set
ii) Closure of a set
iii) Interior of a set

c) Define **5**

i) Connected set
ii) Compact set
iii) Countably compact set.

d) Define **5**

i) T_0 – space
ii) T_1 – space
iii) T_2 – space

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