M.Sc.(Mathematics) (New CBCS Pattern) Semester - I

PSCMTH02 - Paper-II : Real Analysis-I

P. Pages : 2 **GUG/S/23/13738**

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Time : Three Hours Max. Marks : 100

Notes : 1. Solve **all five** questions.
2. Each questions carries equal marks.

UNIT – I

1. a) Prove that the sequence of functions $\{f_n\}$ defined on E, converges uniformly on E if and only if, for every $\in > 0$, there exist an integer N such that $m \geq N$, $n \geq N, x \in E$ implies $|f_n(x) - f_m(x)| < \in$ **10**

b) If K is a compact metric space, $f_n \in C(K)$ for n = 1, 2, 3----- and if $\{f_n\}$ converges uniformly on K then prove that $\{f_n\}$ is equal continuous on K. **10**

OR

c) Prove that, if f is a continuous complex function on [a, b] then there exist a sequence of polynomial p_n such that $\lim\limits_{n\to\infty} p_n(x) = f(x)$ uniformly on [a, b]. **10**

d) For $n = 1, 2, - - - - - x$ real put $f_n(x) = \dfrac{x}{1+nx^2}$ show that $\{f_n\}$ converges uniformly to a function f and that the equation $f'(x) = \lim\limits_{n\to\infty} f_n'(x)$ is correct if $x \neq 0$ but false if x = 0 **10**

UNIT – II

2. a) Prove that suppose f maps an open set $E \subset R^n$ into R^m and f is differentiable at a point $x \in E$ then the partial derivative $(D_j f_i)(x)$ exist and **10**

$$f'(x)e_j = \sum_{i=1}^{m} (D_j f_i)(x) u_i, \ 1 \leq J \leq n$$

b) State and prove Implicit function theorem. **10**

OR

c) If X is complete metric space and if ϕ is the contraction mapping of X into X then prove that there exist $x \in X$ such that $\phi(n) = x$ **10**

d) State and prove the inverse function theorem. **10**

UNIT – III

3. a) Prove that any atlas $\mu = \{U_\alpha, \phi_\alpha\}$ on a locally Euclidean space is contained in a unique maximal atlas. **10**

b) Prove that the following are smooth manifolds. **10**

i) General linear graphs.

ii) Unit circle in the (x, y) plane

OR

c) Let $\{U_\alpha, \phi_\alpha\}$ be an atlas on a locally Euclidean space. If two charts $(\nu, \psi) \& (\omega, \sigma)$ are both compatible with the atlas $\{(U_\alpha, \phi_\alpha)\}$ the prove that they are compatible with each other. **10**

d) Prove that the real line with two origins and sphere with hair are not topological manifolds. **10**

UNIT – IV

4. a) Let M & N be manifolds and $\pi : M \times N \to M$ $\pi(p, q) = p$, the projection to the first factor prove that π is a C^∞ map **10**

b) State and prove the inverse function theorem for manifolds. **10**

OR

c) If (U, ϕ) is a chart on a manifold M of dimension n then prove that the coordinate map $\phi : U \to \phi(U) \subseteq R^n$ is a diffeomorphism. **10**

d) Suppose $F : N \to M$ is C^∞ at $P \in N$. If (U, ϕ) is any chart about P in N and (ν, ψ) is any chart about F(P) in M then prove that $\psi \circ F \circ \phi^{-1}$ is C^∞ at $\phi(P)$. **10**

5. a) If $\{f_n\}$ be a sequence of continuous function on E and $f_n \to f$ uniformly on E then prove that f is continuous on E. **5**

b) Suppose that E open set in R^n $f : E \to R^n$ and $x \in E$ and $\lim_{h \to 0} \frac{|f(x+h) - f(x) - Ah|}{|h|} = 0$ **5**

With $A = A_1$ and $A = A_2$ then prove that $A_1 = A_2$

c) Show that S^1 is a smooth manifold. **5**

d) Define: **5**

i) Smooth function at a point in manifold.

ii) Diffeomorphism.

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