M.Sc.(Mathematics) (New CBCS Pattern) Semester - I

PSCMTH01 - Group Theory & Ring Theory

P. Pages : 2 **GUG/S/23/13737**

Time : Three Hours Max. Marks : 100

Notes : 1. All **five** questions are compulsory.
2. Each question carries equal marks.

UNIT – I

1. a) Let G be a group, and let G' be the derived group of G. Then prove that **10**

i) $G' \triangleleft G$

ii) G/G' is abelian

iii) If $H \triangleleft G$, then G/H is abelian iff $G' \subset H$.

b) Let H & K be normal subgroups of G and $K \subset H$. Then prove that $(G/K)/(H/K) \simeq G/H$ **10**

OR

c) Prove that a nonabelian group of order 6 is isomorphic to S_3. **10**

d) If G is a group with center Z(G), and if G/Z(G) is cyclic, then prove that G must be abelian. **10**

UNIT – II

2. a) Let G be a group. If G is solvable, then prove that every subgroup of G and every homomorphic image of G are solvable. Conversely, if N is a normal subgroup of G such that N & G/N are solvable, then prove that G is solvable. **10**

b) If a permutation $6 \in S_n$ is a product of r transpositions and also a product of S transpositions, then prove that r and S are either both even or both odd. **10**

OR

c) If a cyclic group has exactly one composition series, then prove that it is a p – group. **10**

d) Let G be a nilpotent group. Then prove that every subgroup of G and every homomorphic image of G are nilpotent. **10**

UNIT – III

3. a) Let G be a finite group, and let p be a prime. Then prove that all Sylow P – subgroups of G are conjugate, and their number n_p divides O (G) & satisfies $n_p \equiv 1 (\bmod p)$. **10**

b) Prove that there are no simple groups of orders 42, 48 and 200. **10**

OR

c) Let G be a finite group, and let p be a prime. If p^m divides $|G|$, then G has a subgroup of order p^m. **10**

d) Let G be a group of order pq, where p & q are prime numbers such that $p > q$ and $q \nmid (p-1)$. Then prove that G is cyclic. **10**

UNIT – IV

4. a) For any two ideals A & B in a ring R, prove that **10**

i) $\dfrac{A+B}{B} \simeq \dfrac{A}{A \cap B}$

ii) $\dfrac{A+B}{A \cap B} \simeq \dfrac{A+B}{A} \times \dfrac{A+B}{B} \simeq \dfrac{B}{A \cap B} \times \dfrac{A}{A \cap B}$

b) If a ring R has unity, then prove that every ideal I in the matrix ring R_n is of the form A_n, where A is some ideal in R. **10**

OR

c) For any ring R and any ideal $A \neq R$, prove that the following are equivalent: **10**
i) A is maximal
ii) The quotient ring R/A has no nontrivial ideals.
iii) For any element $x \in R,\ x \notin A,\ A + (x) = R$.

d) Let R be a commutative principal ideal domain with identity. Then prove that any nonzero ideal $P \neq R$ is prime if & only if it is maximal. **10**

5. a) Define: **5**
i) Normal subgroup
ii) Commutator subgroup

b) Define: **5**
i) Alternating group
ii) Solvable group

c) Prove that a group of order 1986 is not simple. **5**

d) Define: **5**
i) Ideal
ii) Ring homomorphism.

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