M.Sc. (Mathematics) (NEW CBCS Pattern) Sem-I

PSCMTH01: Group Theory & Ring Theory

P. Pages : 2 **GUG/W/22/13737**

Time : Three Hours * 3 4 4 1 * Max. Marks : 100

Notes : 1. All **five** questions are compulsory.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that every group is isomorphic to a permutation group. **10**

b) Let $\phi : G \to G'$ be a homomorphism of groups. Then prove that $G / \ker \phi \simeq \text{Im}\phi$. **10**

OR

c) Prove that the set Aut (G) of all automorphisms of a group G is a group under composition of mappings, and In (G) $\triangleleft \text{Aut}(G)$. Moreover, prove that $G / Z(G) \simeq \text{In}(G)$. **10**

d) Let G be a finite group acting on a finite set x. Then prove that the number k of orbits in X under G is **10**

$$K = \frac{1}{|G|}\sum_{g \in G} |X_g|$$

UNIT – II

2. a) Prove that any two composition series of a finite group are equivalent. **10**

b) Prove that the alternating group An is simple if $n > 4$. Moreover, prove that S_n is not solvable if $n > 4$. **10**

OR

c) Let G be a nilpotent group. Then prove that every subgroup of G and every homomorphic image of G are nilpotent. **10**

d) Prove that any two conjugate permutations in S_n have the same cycle structure. Conversely, prove that any two permutations in S_n with the same cycle structure are conjugate. **10**

UNIT – III

3. a) Let G be a finite group, and let p be a prime. If p^m divides |G|, then prove that G has a subgroup of order p^m. **10**

b) Prove that there are no simple groups of orders 63, 56, and 36. **10**

OR

c) Let G be a group of order pq, where p & q are prime number such that $p > q$ & $q \nmid (p-1)$. Then prove that G is cyclic. **10**

d) Let G be a group of order 108. Prove that there exists a normal subgroup of order 27 or 9. **10**

UNIT – IV

4. a) If a ring R has unity, then prove that every ideal I in the matrix ring R_n is of the form A_n, where A is some ideal in R. **10**

b) In a nonzero commutative ring with unity, prove that an ideal M is maximal if & only if R/M is a field. **10**

OR

c) Let f be a homomorphism of a ring R into a ring S with kernel N. Then prove that $R / N \simeq \text{Im} f$. **10**

d) If R is a nonzero ring with unity 1 and I is an ideal in R such that $I \neq R$ then prove that there exists a maximal ideal M of R such that $I \subseteq M$. **10**

5. a) If G is a group and H is a subgroup of index 2 in G, then prove that H is a normal subgroup of G. **5**

b) Prove that the derived group of S_n is A_n. **5**

c) If the order of group is 42, prove that its sylow 7- subgroup is normal. **5**

d) Define **5**
i) Maximal Ideal &
ii) Prime ideal.

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