# 021C - Mathematics Paper-IV - DSE-VIII : Special Relativity-II

P. Pages : 2 **GUG/W/22/13362**

Time : Three Hours Max. Marks : 60

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Notes : 1. All the questions are compulsory.
2. Each question carries equal marks.

## UNIT – I

**1.** a) If $f = a_{rs} x^r x^s$ then show that **6**

$$\frac{\partial f}{r x^r} = \left(a_{rs} + a_{sr}\right) x^s, \frac{\partial^2 f}{\partial x^r \partial x^s} = a_{rs} + a_{sr}$$

b) A covariant vector has components $2x - z,\ x^2 y,\ yz$ in rectangular coordinates find its covariant components in cylindrical coordinates. **6**

**OR**

c) For a mixed tensor $T^m_{nrs}$ of order 4 show that $T^n_{nrs}$ is a tensor of order 2. **6**

d) Show that **6**

i) $dg_{pr} = -g_{ps} g_{rm} dg^{sm}$

ii) $\dfrac{dg}{g} = g^{mn} dg_{mn} = -g_{mn} dg^{mn}$.

## UNIT – II

**2.** a) Show that: **6**

i) $[mn, r] = g_{rs} \Gamma^s_{mn}$

ii) $g_{mn,r} = [mr, n] + [nr, m]$

b) Find nonvanishing components of Christoffel symbols of 2nd kind for **6**

$$ds^2 = \left(dx^1\right)^2 + f\left(x^1, x^2\right)\left(dx^2\right)^2.$$

**OR**

c) Show that under a linear transformation of a coordinate system $x^m = a^m_n x'^n + b^m$ the Christoffel symbols are tensors, where $a^m_n$ & $b^m$ are constants. **6**

d) Show that the absolute derivative of a scalar is its ordinary derivative **6**

ie $\dfrac{\delta A}{\delta u} = \dfrac{dA}{du}$, where A is scalar .

## UNIT – III

**3.** a) Obtain the mass of the moving particle. **6**

b) Show that $P^2 - E^2/C^2$ is invariant whose numerical value is $-m_0^2 c^2$. **6**

**OR**

c) Show that the four velocity in component form can be expressed as- **6**

$$u^i = \left( \frac{\bar{u}}{C\sqrt{1-u^2/c^2}}, \frac{1}{\sqrt{1-u^2/c^2}} \right), \bar{u} = \left(u_x, u_y, u_z\right)$$

d) Obtain the equation of motion of a free particle. **6**

## UNIT – IV

**4.** a) Obtain the Maxwell's equations of electromagnetic theory in vacuum in the component form. **6**

b) Show that the Hamiltonian for a charged particle moving in an electromagnetic field is **6**

$$H = \left[ m_0^2 c^4 + c^2 \left( p - \frac{e}{c} A \right)^2 \right]^{\frac{1}{2}} + e\phi .$$

**OR**

c) An electromagnetic field is purely magnetic in an inertial frames then describe the field in inertial frame's. **6**

d) Explain the gauge transformations. **6**

**5.** Solve **any six.**

a) Show that $a_{mn} x^m x^n = 0$ for a skew symmetric tensor $a_{mn}$. **2**

b) Define covariant tensor of order 1 & 2. **2**

c) Show that $R_{rm}$ is a symmetric tensor. **2**

d) Define Einstein tensor. **2**

e) Show that the four force & four velocity are orthogonal to each other. **2**

f) Show that $\frac{dE}{dp} = u$. **2**

g) Write the equation $E = -\text{grod}\phi - \frac{1}{c}\frac{\partial A}{\partial t}$ in component form. **2**

h) Show that the energy momentum tensor $T^{ij}$ is symmetric. **2**

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