B.Sc. (CBCS Pattern) Sem-VI

021A - Mathematics-II - DSE-VI : Complex Analysis and Vector Calculus

P. Pages : 2 **GUG/W/22/13361**

Time : Three Hours * 2 2 9 2 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each questions carry equal marks.

UNIT – I

1. a) If F(z) is analytic function with constant modulus, show that F(z) is constant. **6**

b) Prove that the function sin z is analytic and find it's derivative. **6**

OR

c) Show that $u = 2x - x^3 + 3xy^2$ is harmonic and find it's harmonic conjugate function. Hence find the analytic function F(z) = u + iv. **6**

d) Prove that the cross ratio remains invariant under a bilinear transformation. **6**

UNIT – II

2. a) Evaluate $\int_0^{1+2i} z^{-2}\, dz$ along the real axis to 1 and then vertically to 1 + 2i. **6**

b) Let F(z) be analytic in a simply connected domain D and let C be a simple closed curve in D oriented counterclockwise then prove that for any point a within C, **6**

$$\oint_C \frac{F(z)}{z-a}\, dz = 2\pi i\, f(a)$$

OR

c) Let z = a be a simple poles of $F(z) = \frac{F(z)}{G(z)}$, such that $F(a) \neq 0$. Then prove that **6**

$$R_a = \frac{F(a)}{G'(a)}.$$

d) Evaluate the residue of $f(z) = \frac{z^2}{(z-1)(z-2)\,(z-3)}$ at z = 1, 2, 3 and infinity and show that their sum is zero. **6**

UNIT – III

3. a) If $\phi = 3x^2y - y^3z^2$, find grad ϕ at the point $(1, -2, -1)$ **6**

b) Find the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ at (1, 1, -1) in the directional of the vector $2i + j - k$, In what directional derivative from the point $(1,1,-1)$ is maximum and what is it's value? **6**

OR

c) If $\overline{F} = x^2zi - 2y^3z^2j + xy^2zk$, find div $\overline{F}$ and curl $\overline{F}$ at $(1,-1,1)$. **6**

d) The acceleration of a particle at any time is $e^t i + e^{2t} j + k$. Find the velocity $\overline{v}$ and r if v and r are zero at t = 0. **6**

UNIT – IV

4. a) Evaluate $\iint_S \overline{f}.n\,ds$, where $\overline{f} = x^2i + y^2j + z^2k$ and S is the surface of the solid cut off by the plane x + y + z = a from the first octant. **6**

b) Evaluate by stokes theorem $\int_c (e^x dx + 2ydy - dz)$, where C is the curve $x^2 + y^2 = 4,\ z = 2$ **6**

OR

c) State and prove stokes' theorem. **6**

d) Verify Green's theorem in the plane for $\oint_c (3x^2 - 4y^2)\,dx - 2xydy$, where C is the boundary of the region R defined by $y = \sqrt{x},\ y = x^2$. **6**

5. Solve **any six** of the following.

i) State Cauchy-Riemann equation in cartesian form. **2**

ii) Let F(z) = x show that the CR equation are not satisfied in the z-plane. **2**

iii) Define simple closed curve. **2**

iv) Evaluate $\int_C (z - z^2)\,dz$, where C is the upper half of the circle $|z| = 1$. **2**

v) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$, find div $\overline{r}$ and curl $\overline{r}$ **2**

vi) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$. **2**

vii) Define Simply connected regions R. **2**

viii) State Green's theorem in a plane. **2**

****