B.Sc.- III (CBCS Pattern) Sem-V

USMT11 - Mathematics Paper-III - DSE : Matrices and Theory of Equations

P. Pages : 2 **GUG/W/22/13117**

Time : Three Hours * 3 7 6 5 * Max. Marks : 60

Notes : 1. Solve all **five** question.
2. Que 1 to 4 has an alternative. Solve each question in full or its alternative in full.
3. All question carry equal marks.

UNIT – I

1. a) If $A=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$ then show that A is orthogonal matrix. **6**

b) Let $A=\begin{bmatrix}3 & 2 & -1\end{bmatrix}_{1\times 3}$ and $B=\begin{bmatrix}0\\ -1\\ 3\end{bmatrix}_{3\times 1}$ are two matrices then show that $AB \neq BA$ **6**

OR

c) For the matrix $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & -1 & -1\end{bmatrix}$, find the nonsingular matrices P and Q such that PAQ is in the normal form. **6**

d) Find the rank of matrix $A=\begin{bmatrix}1 & 3 & 4 & 3\\ 3 & 9 & 12 & 9\\ 1 & 3 & 4 & 0\end{bmatrix}$. **6**

UNIT – II

2. a) If X_1 and X_2 are solution of system of homogenous linear equations $AX=0$, then prove that their linear combination is a solution of $AX=0$. **6**

b) Show that equations $2x+6y=-11,\ 6x+20y-6z=-3,\ 6y-18z=-1$ are not consistent. **6**

OR

c) Solve the following equations by Cramer's rule. **6**
$x+y+z=4,\ x-y+z=0,\ 2x+y+z=5$

d) Show that if B is an invertible matrix of same order as A, then show that matrix A and $B^{-1}AB$ have the same characteristics root. **6**

UNIT – III

3. a) Prove that every equation of degree n has n roots and no more. **6**

b) Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$, roots being in arithmetical progression. **6**

OR

c) Prove that the negative roots of $f(x)$ are the positive roots of $f(-x)$. **6**

d) Find the equation whose roots are the roots of $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 4. **6**

UNIT – IV

4. a) Find the equation whose roots are the negative reciprocals of the roots if $x^4 + 7x^3 + 8x^2 - 9x + 10 = 0$ **6**

b) If α, β, γ are the roots of $x^3 - 2x + 5 = 0$, form an equation whose roots are $\frac{\alpha}{\beta+\gamma-\alpha}, \frac{\beta}{\gamma+\alpha-\beta}, \frac{\gamma}{\alpha+\beta-\gamma}$ and find the value of $\sum \frac{\alpha}{\beta+\gamma-\alpha}$. **6**

OR

c) Solve the equation $x^3 - 21x = 344$ by Cardon's method. **6**

d) Solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ **6**

5. Solve **any six.**

a) $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ then find A^T. **2**

b) Define a nonsingular matrix. **2**

c) Define inconsistent system of linear equations. **2**

d) State the Caley – Hamilton theorem for the matrix. **2**

e) Find the nature of the roots of the equation $3x^4 + 12x^2 + 5x - 4 = 0$. **2**

f) From the rational cubic equation of lowest degree which shall have the roots 1, 3 – 2i. **2**

g) Write a general form of biquadratic equation. **2**

h) State the Descartes's rule of sign for roots of equation $f(x) = 0$. **2**

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