B.Sc.-III (CBCS Pattern) Semester - V

USMT10 - DSE - Mathematics-II (Mechanics)

P. Pages : 2 **GUG/S/23/13116**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Obtain the tangential and normal components of the velocity and acceleration of the moving particle in plane. **6**

b) A particle describes an equiangular spiral $r = a e^{m\theta}$ with constant velocity. Find the components of the velocity and of acceleration along the radius vector and perpendicular to it. **6**

OR

c) A point moves along the arc of a cycloid in such a manner that the tangent at it rotates with a constant angular velocity, show that the acceleration of the moving point is constant in magnitude. **6**

d) A particle is moving with simple harmonic motion and while making an excursion from one position of rest to another, its distance from the middle point of its path as three consecutive seconds are observed to be x_1, x_2 and x_3. Prove that the time of complete oscillation is $2\pi / \cos^{-1}\left(\frac{x_1 + x_3}{2x_2}\right)$ **6**

UNIT – II

2. a) Prove that total angular momentum of the system of particles about any point O is the sum of the angular momentum of the system concentrated at the centre of mass and angular momentum of motion about the centre of mass. **6**

b) Prove that total K.E. of the system is the sum of the K.E. of the total mass concentrated at the centre of mass G and K.E. of the system about the center of mass G. **6**

OR

c) If the external and internal forces are both conservative, then prove that total potential energy V can be expressed as $V = \sum_{i} V_i + \frac{1}{2}\sum_{i,j} V_{ij}$. **6**

d) Prove that the magnitude R of the position vector for the center of mass from an arbitrary origin is given by $M^2R^2 = M\sum_{i} m_i r_i^2 - \frac{1}{2}\sum_{i,j} m_i m_j r_{ij}^2$. **6**

UNIT – III

3. a) Derive the Lagranges equation of motion $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$ for conservative system from D'Alembert's principle. **6**

b) Construct the lagrangian for a particle moving in space and then deduce the equation of motion by using cartesian co-ordinates. **6**

OR

c) Two particles of masses m_1 and m_2 are connected by a light inextensible string which passes over a small smooth fixed pulley if $m_1 > m_2$, then show that the common acceleration of a particles is $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$. **6**

d) Prove that the rate of energy dissipation due to friction is 2R. **6**

UNIT – IV

4. a) Show that the problem of motion of two masses interacting only with one another always be reduced to a problem of the motion of a single mass. **6**

b) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force field. Find the law of force. **6**

OR

c) Prove that the square of the periodic time of the planet is proportional to the cube of the semi-major axis of its elliptic orbit. **6**

d) Prove that for a system moving in a finite region of space with finite velocity, the time average of kinetic energy is equal to the virial of the system. **6**

5. Attempt **any six.**

a) Write the radial transverse components of velocity of a moving particle in a circular path of radius a. **2**

b) If radial and transverse velocities of a particle are always proportional to each other, then find path of moving particle. **2**

c) State Newton's second law of motion. **2**

d) If the total torque on the particle is zero, then prove that the angular momentum is conserved. **2**

e) Define degree of freedom. **2**

f) Define Rayleigh's dissipation function. **2**

g) State Kepler's first law of planetory motion. **2**

h) Prove that the path of a particle in a central force field lies in one plane. **2**

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