B.Sc. (CBCS Pattern) Sem-V

USMT10 - Mathematics DSE-II : Mechanics

P. Pages : 2 **GUG/W/22/13116**

Time : Three Hours * 1 2 2 5 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. Each questions carries equal marks.

UNIT- I

1. a) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr^2 and $\mu\theta^2$. Show that the equation to the path is $\frac{\lambda}{\theta} = \frac{\mu}{2r^2} + C$ and component accelerations are $2\lambda^2 r^3 - \mu^2 \frac{\theta^4}{r}$ and $\lambda\, \mu\, r\, \theta^2 + 2\mu^2 \frac{\theta^3}{r}$ **6**

b) A particle moves along a circle $r = 2a\, \cos\theta$ in such a way that its acceleration towards the origin is always zero. **6**
Prove that
$\frac{d\omega}{dt} = -2\omega^2 \cot\theta$ where $\omega = \frac{d\theta}{dt}$

OR

c) Prove that if the tangential and normal acceleration of a particle describing a plane curve be constant throughout the motion, the angle ψ which the direction of motion turns in time t is given by **6**
$\Psi = a\, \log(1 + Bt)$

d) The position of a particle moving in a straight line is given by $x = a\cos nt + b\sin nt$. Prove that it executes SHM of period ${}^{2\pi}/_{n}$ and amplitude $\sqrt{\left(a^2 + b^2\right)}$ **6**

UNIT – II

2. a) If the forces acting on the particle are conservative, then prove that the total energy of the particle is conserved. **6**

b) Prove that the centre of mass of the particles moves as if the external forces were acting on the mass of the system concentrated at the centre of mass. **6**

OR

c) If the total external force on the system of particles is zero then prove that the total linear momentum is conserved. **6**

d) Prove that the magnitude R of the position vector for the centre of mass from an arbitrary origin is given by **6**

$$M^2 R^2 = M \sum_i m_i \bar{r}_i^{\,2} - \frac{1}{2} \sum_{i,j} m_i\, m_j\, \bar{r}_{ij}^{\,2}$$

UNIT – III

3. a) Prove that the virtual work on a mechanical system (for which the net virtual work of the forces of constraint vanishes) by the applied forces and the reversed effective forces is zero. **6**

b) If L is a Lagrangian for a system of n degrees of freedom satisfying Lagrange's equations, show by direct substitution that $L' = L + \frac{dF}{dt}$, $F = F(q_1, q_2, ---q_n, t)$ also satisfies Lagrange's equations. Where F is any arbitrary but differentiable function of its argument. **6**

OR

c) A particle moves in a plane under the influence of a force acting towards a centre of force whose magnitude is $F = \frac{1}{r^2}\left[1 - \frac{\dot{r}^2 - 2\ddot{r}\,r}{c^2}\right]$, where r is the distance of the particle to the centre of force. Find the generalized potential that will result in such a force. **6**

d) Prove that the rate of energy dissipation due to friction is 2R, where R is the Rayleigh's dissipation function. **6**

UNIT – IV

4. a) Prove that the problem of motion of two masses interacting only with one another always be reduced to a problem of the motion of single mass. **6**

b) Prove that for a central force field F, the path of a particle of mass m is given by **6**

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{h^2u^2} F\left(\tfrac{1}{u}\right), \qquad u = \frac{1}{r}$$

OR

c) For a central force field, show that Kepler's second law is a consequence of the conservation of angular momentum. **6**

d) For a system moving in a finite region of space with finite velocity, the time average of K.E. is equal to the virial of the system. **6**

5. Solve **any six.**

a) Find the radial components of acceleration of a moving particle in a circular path radius a. **2**

b) Define frequency **2**

c) Prove that $\overline{N} = \dot{\overline{M}}$ **2**

Where $\overline{N}$ is moment of force & $\overline{M}$ is angular momentum.

d) Write definition of centre of mass. **2**

e) Prove that the force of constraint does no work in any possible displacement. **2**

f) Write Lagrange's equations of motion for conservative system from D'Alembert's principle. **2**

g) Prove that a central force motion is a motion in a plane. **2**

h) Define potential well. **2**

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