B.Sc.- III (CBCS Pattern) Sem-V

USMT09 - Mathematics Paper-I - DSE : Linear Algebra

P. Pages : 2 **GUG/W/22/13115**

Time : Three Hours Max. Marks : 60

Notes : 1. Solve **all the five** questions.
2. Question No. 1 to 4 has an alternative. Solve each question in full or its alternative in full.
3. All questions carry equal marks.

1. a) Let V be a vector space over F. Then prove that **6**
i) $\alpha 0 = 0,\ \forall \alpha \in F$
ii) $0V = 0,\ \forall v \in V$
iii) $(-\alpha)V = -(\alpha V),\ \forall \alpha \in F, \forall v \in V$

b) Let U, W be subspace of vector space V(F). Prove that $U \vee W$ is subspace of V iff $U \subseteq W$ or $W \subseteq U$. **6**

OR

c) If $V_1, V_2, V_3, ..., V_n$ is a basis of V or span V over F and $W_1, W_2, ..., W_m \in V$ are linearly independent over F, then prove that $m \le n$. **6**

d) Let W be subspace of a finite dimensional vector space V. Then prove that W is finite dimensional. **6**

2. a) Let U, V be vector spaces over the same field F. Then prove that a function $T : U \to V$ is linear iff $T(\alpha u + v) = \alpha \cdot T(u) + TV\ \ \forall\ u, v \in \vee$ and $\alpha \in F$. **6**

b) Let a mapping $T : V_2 \to V_2$ be defined by $T(x, y) = (x', y')$ where $x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$. Then show that T is linear map. **6**

OR

c) Let $T : U \to V$ be linear map, then prove that R(T) is subspace of V. **6**

d) Show that the linear map $T : V_3 - V_3$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, x_2 + x_3, x_3)$ is non singular and find its inverse. **6**

3. a) Let V be a finite dimensional vector space over F and $V(\neq 0) \in V$. Then prove that there exists $f \in \hat{V}$ such that $f(V) \neq 0$. **6**

b) For a finite dimensional vector space V, Show that $\hat{V}$ is a separating family. **6**

OR

c) Let $T: R^2 \to R^2$ be linear map defined by $T(x, y) = (0, y)$. If $\lambda \in R$ is on eigen value of T, then $\exists (x, y) \neq (0, 0)$ in R^2 such that $T(x, y) = \lambda(x, y)$ find the value of λ and eigen vector of T. **6**

d) Let V be the space of all real valued continuous function of real variable. Define $T: V \to V$ by $(Tf)(x) = \int_0^x f(t)\,dt, \forall f \in V, x \in R$. Show that T has no eigen value. **6**

4. a) Let V be set of all continuous complex – valued functions on the closed interval [0, 1]. If $f(t), g(t) \in V$, define $(f(t), g(t)) = \int_0^1 f(t)\,\bar{g}(t)\,dt$. Show that this defines inner product on V. **6**

b) Let V be an inner product space over F. if $u, v \in V$ then prove that **6**

$$|(u, v)| \le \|u\| \cdot \|v\|$$

OR

c) Let $\{x_1, x_2, x_3, ..., x_n\}$ be orthogonal set. Then show that **6**

$$\|x_1 + x_2 + ... + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + ...\|x_n\|^2$$

d) If V be a finite dimensional inner product space and W is subspace of V then prove that $(W^{\perp})^{\perp} = W$. **6**

5. Solve **any six.**

a) Let V be a vector space over F. Then prove that $0V = 0, \ \forall\ v \in V$. **2**

b) Define a basis of a vector space. **2**

c) Show that $T: R^2 \to R^2$, defined by $T(x, y) = (x + 1, y + 2)$ is not a linear map. **2**

d) Define a Null space in vector space. **2**

e) Define a dual space of vector space V. **2**

f) Let $\lambda \neq 0$ be an eigen value of an invertiable linear transformation $T \in L(V)$. Show that λ^{-1} is an eigen value of T^{-1}. **2**

g) Define a orthogonal complement in inner product space. **2**

h) Prove that in inner product space **2**

$$W \cap W^{\perp} = \{0\}$$

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