B.Sc. (CBCS Pattern) Semester - V

USMT09 - DSE-I - Mathematics-I (Linear Algebra)

P. Pages : 2 **GUG/S/23/13115**

0258

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Prove that, a nonempty subset U of a vector space V over field F is a subspace of V iff **6**
i) $u+v \in U, \forall u, v \in U$ ii) $\alpha u \in U \ \forall \ \alpha \in F, \ u \in U$

b) Prove that an arbitrary intersection of subspaces of a vector space is a subspace. **6**

OR

c) Prove that the set $\beta=\{(1,1,1)(1,-1,1)(0,1,1)\}$ is a basis of V_3. **6**

d) Let W be a subspace of finite dimensional vector space V. Then prove that W is finite dimensional. **6**

UNIT - II

2. a) Let U, V be vector spaces over a field F and $T: U \to V$ be a linear map. Then prove that **6**
i) $T(0)=0$
ii) $T(-u)=-Tu \ \forall n \in U$
iii) $T(\alpha_1 u_1 + \alpha_2 u_2 + \ldots\ldots + \alpha_n u_n) = \alpha_1 Tu_1 + \alpha_2 Tu_2 + \ldots\ldots + \alpha_n T(u_n)$
$\forall u_i \in U, \ \alpha_i \in F, \ i=1,2....n \ n \in N$.

b) Let a mapping $T: V_2 \to V_2$ be defined by $T(x,y)=(x', y')$ where **6**
$x' = x\cos\theta - y\sin\theta, \ y' = x\sin\theta + y\cos\theta$ show that T is a linear map.

OR

c) Let $T: U \to V$ be a linear map then prove that **6**
i) R (T) is a subspace of V ii) N (T) is a subspace of U

d) Show that the linear map $T: v_3 \to v_3$ defined by **6**
$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, \ x_2 + x_3, \ x_3)$ is non singular and find its inverse.

UNIT - III

3. a) Let V be the finite dimensional vector space over F then prove that $V \approx \hat{\hat{V}}$. **6**

b) Let w_1 and w_2 are two subspaces of finite dimensional vector space V then prove that **6**
$A(w_1 + w_2) = A(w_1) \cap A(w_2)$ where $A(w_1)$ and $A(w_2)$ are annihilator of w_1, w_2.

OR

c) Let V be the vector space of all real valued continuous function of real variable. **6**

Define $T: V \rightarrow V$ by $(TF)(x) = \int_0^x f(t)\,dt, \forall f \in v,\ x \in R$. Show that T has no eigen value.

d) Prove that the element $\lambda \in f$ is CR of $T \in L(\nu)$ iff for some $\nu(\nu \neq 0) \in v,\ T_v = \lambda_v$. **6**

UNIT - IV

4. a) Let V be an inner product space over F. If $u, v, \in V$ then prove that $|(u,v)| \le \|u\|\ \|v\|$. **6**

b) Let $\{x_1, x_2, ...x_n\}$ be an orthogonal set then prove that $\|x_1 + x_2 + ... + x_n\|^2 = \|x_1\|^2 + \|x_2\|^2 + ... + \|x_n\|^2$. **6**

OR

c) If $\{w_1, w_2, ...w_m\}$ is an orthonormal set in V then prove that **6**

$$\sum_{i=1}^{m} \left|\left(w_{1,} v\right)\right|^2 \le \|v\|^2 \text{ for } \nu \in V$$

d) Using Gram-Schmidt orthogonalization process, orthonormalize the L.I. subset $\{(1,1,1), (0,1,1), (0,0,1)\}$ of V_3. **6**

5. Solve **any 6** questions.

a) Let V be a vector space over F then prove that $\alpha \cdot 0 = 0, \forall \alpha \in F$. **2**

b) If S and T are subsets of a vector space V then prove that $S \subseteq T \Rightarrow L(s) \subseteq L(T)$. **2**

c) Let $T: U \rightarrow V$ be a linear map then prove that T is one-one $\Leftrightarrow N(T)$ is a zero subspace of U. **2**

d) Let $T: V_2 \rightarrow V_3$ be a linear map defined by $T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$ show that T is 1-1. **2**

e) Define a second dual vector space. **2**

f) Let $\lambda \neq 0$ be an eigen value of an invertible linear transformation T show that λ^{-1} is an eigen value of T^{-1}. **2**

g) Prove that $W \cap W^1 = \{0\}$. **2**

h) Prove that, If V is a inner product space over F then. $(u, \alpha v + \beta w) = \bar{\alpha}(u,v) + \bar{\beta}(u,w) \forall u, v, w \in V\ \alpha\ \beta \in F$. **2**

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