B.Sc. (CBCS Pattern) Sem-IV

USMT08 - Mathematics-II Paper-VIII (Elementary Number Theory)

P. Pages : 2 GUG/W/22/12015

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that, the product of any m consecutive integers is divisible by m! **6**

b) Prove that $3/n\,(n+1)(2n+1),\quad A \in Z$. **6**

OR

c) Prove that, no integers x, y exist satisfying x + y = 100 and (x, y) = 3. **6**

d) Find the g.c.d. of 275 and 200 and express it in the form 275 x + 200y. **6**

UNIT – II

2. a) Prove that every positive integer greater than one has at least one prime divisor. **6**

b) Find the g. c.d. and ℓ.c.m. of a = 18900 and b = 17160 by writing each of the numbers a and b in prime factorization canonical form. **6**

OR

c) Prove that, for all positive integers n, $F_0, F_1 .... F_{n-1} = F_n - 2$. **6**

d) Prove that, the Fermat number F_5 is divisible by 641 and hence is composite. **6**

UNIT – III

3. a) Let $a_1, a_2, b_1, b_2 \in Z$ such that $a_1 \equiv b_1 (\mathrm{mod}\, m)$ and $a_2 \equiv b_2 (\mathrm{mod}\, m)$, then prove that **6**

i) $(a_1 + a_2) \equiv (b_1 + b_2)(\mathrm{mod}\, m)$

ii) $a_1 \cdot a_2 \equiv b_1 \cdot b_2\ (\mathrm{mod}\, m)$

iii) $c \cdot a_1 \equiv c \cdot b_1\ (\mathrm{mod}\, m),\ c \in z$

b) Prove that, if $a \equiv b (\mathrm{mod}\, m)$ then $a^n \equiv b^n\ (\mathrm{mod}\, m), \forall\, n \in N$ **6**

OR

c) Find all solutions of $15x \equiv 12 \pmod{g}$. **6**

d) Solve the system of three congruences $x \equiv 1 \pmod{4},\ x \equiv 0 \pmod{3},\ x \equiv 5 \pmod{7}$. **6**

UNIT – IV

4. a) Find all the positive integers x and y such that $x^{\phi(y)} = y$. **6**

b) Solve linear congruence $4x \equiv 7 \pmod{15}$ **6**

OR

c) Prove that, if x, y, z is Pythagorean triple and (x, y) = d then (y, z) = (z, x) = d. **6**

d) Show that the integer solution of $x^2 + 2y^2 = z^2$ with (x, y, z) = 1 can be expressed as $x = \pm(2a^2 - b^2), y = 2ab, z = 2a^2 + b^2$. **6**

5. Solve **any six**:

a) Evaluate $[n, n+1]$, where $n \in N$. **2**

b) Find (5325, 492) **2**

c) Define Fermat Numbers. **2**

d) Prove that $(a^2, b^2) = c^2$ if (a, b) = c. **2**

e) Let a, b, c be integer such that $a \equiv b$ (mod m) then prove that $ac \equiv bc \pmod{mc}, c > 0$ **2**

f) Find the remainder when 43^{289} is divided by 7. **2**

g) Define Pythagorean Triple. **2**

h) Prove that, if x, y, z is a primitive Pythagorean triple in which the difference z – y = 2 then x = 2t, $y = t^2 - 1,\ z = t^2 + 1$, for some t > 1. **2**

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