B.Sc. (Part-II) (CBCS Pattern) Sem-IV

USMT07 - Mathematics-I Paper-VII : Algebra

P. Pages : 2 **GUG/W/22/12014**

Time : Three Hours Max. Marks : 60

* 2 3 8 9 *

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Show that if G is an abelian group, then $(ab)^n = a^n b^n \; \forall a, b \in G$ and $\forall$ integers n. **6**

b) If H and K are subgroups of G, show that $H \cap K$ is a subgroup of G. **6**

OR

c) Prove that- **6**
$(12345678)^4 = (15)(26)(37)(48)$

d) For $S = \{1, 2, 3, ...9\}$ and $a, b \in A(S)$, compute $a^{-1}ba$ where $b = (1579)$, $a = (135)(12)$. **6**

UNIT - II

2. a) Show that any two right cosets of a subgroup are either disjoint or identical. **6**

b) If G is a finite group and H is a subgroup of G, then prove that O(H) is a divisor of O(G). **6**

OR

c) Prove that, **6**
N is normal subgroup of G if and only if $gNg^{-1} = N, \; \forall g \in G$.

d) Suppose that N and M are two normal subgroups of G and that $N \cap M = (e)$, show that for any $n \in N, \; m \in M, \; nm = mn$. **6**

UNIT - III

3. a) If ϕ is an homomorphism of a group G into a group G', then prove that- **6**

i) $\phi(e) = e^1$

ii) $\phi\left(x^{-1}\right) = (\phi(x))^{-1} \; \forall x \in G$

b) Let N be a normal subgroup of G. Define the mapping $\phi : G \to G / N$ such that $\phi(x) = Nx \; \forall x \in G$. Then show that ϕ is a homeomorphism of G onto G/N. **6**

OR

c) If ϕ is a homormophism of G into G' with Kernel K then prove that K is normal subgroup of G. **6**

d) Let ϕ be a homormophism of G onto G' with kernel K. Then prove that $G/K \approx G'$. **6**

UNIT - IV

4. a) Let R be a ring with unit element, 1, in which $(ab)^2 = a^2 b^2 \; \forall \, a, b \in R$. Prove that R must be commutative. **6**

b) Prove that A nonempty subset S of a ring R is a subring of $R \Leftrightarrow x - y, \; xy \in s \forall x, y \in s$. **6**

OR

c) Show that the ring $R = \left\{a + b\sqrt{2} \big/ a, b \in Z\right\}$ is an integral domain under addition and multiplication. **6**

d) Prove that the set of units in a commutative ring with unity is a multiplicative abelian group. **6**

5. Solve **any six.**

a) Let G be a group. Then show that every $a \in G$ has a unique inverse in G. **2**

b) Define even and odd permutation. **2**

c) Define right coset and left coset of subgroup H in group G. **2**

d) Define; Quotient Group. **2**

e) Define homormophism and isomorphism. **2**

f) G is a group of real numbers under addition $\phi : G \to G$ such that $\phi(x) = 13x \forall x \in G$ then prove that ϕ is homomorphism. **2**

g) Define Ring. **2**

h) Define field. **2**

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