USMT-06 - Mathematics-II Paper-VI : Set Theory and Laplace Transform

P. Pages : 2

Time : Three Hours

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GUG/S/23/11613

Max. Marks : 60

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Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that $(A \cup B)' = A' \cap B'$. **6**

   b) Show that if $a, b \in R$ then there is one & only one number x such that $x + a = b$. This number x is given by $x = b + (-a)$ **6**

   OR

   c) Show that $x \in R \Rightarrow x^2 \ge 0$. **6**

   d) Prove that Let $\langle A_n \rangle$ be a sequence of countable sets. Then $S = \bigcup_{n=1}^{\infty} A_n$ is countable. **6**

UNIT – II

2. a) Prove that A fuzzy set F on the universal set R is convex if and only if $F[\lambda x_1 + (1-\lambda) x_2] \ge \min[F(x_1), F(x_2)] \ \forall x_1, x_2 \in R$ & all $\lambda \in [0,1]$. **6**

   b) Find $\tilde{A}\tilde{B}$, where **6**

   $\tilde{A} = \dfrac{0.9}{1} + \dfrac{0.7}{3} + \dfrac{0.2}{4} + \dfrac{0.3}{6}$,

   $\tilde{B} = \dfrac{0.1}{2} + \dfrac{0.4}{3} + \dfrac{0.5}{4} + \dfrac{0.8}{5}$ are defined on the universe $U = \{1, 2, 3, 4, 5, 6\}$

   OR

   c) Let $\tilde{A}, \tilde{B} \in P(U)$ Then for all $\alpha \in [0,1]$ show that **6**

   i) $\tilde{A} = \tilde{B}$ if and only if $^{\alpha}\tilde{A} = {}^{\alpha}\tilde{B}$

   ii) $\tilde{A} = \tilde{B}$ if and only if $^{\alpha+}\tilde{A} = {}^{\alpha+}\tilde{B}$

   d) Let $\tilde{A} \in P(U)$ then prove that $^{\alpha}(A') = [^{(1-\alpha)^+} A]', \ \forall \alpha \in [0,1]$. **6**

UNIT – III

3. a) Show that if $f(t)$ is piecewise continuous in every finite interval $0 \le t \le T$ for every $T > 0$ & of exponential order α for $t > T$ then its Laplace Transform $F(s)$ exists $\forall S > \alpha$. **6**

   b) Find the Laplace Transform of function $\cosh at \cdot \sin at$. **6**

   OR

c) Find Laplace transform of **6**

$$\cosh t \int_0^t e^t \cosh t \, dt$$

d) Obtain the Laplace Transform of the **6**

$$x'' + 2x' - 3x, \ x(0) = 0, \ x'(0) = 1$$

UNIT – IV

4. a) Find the solution by Laplace Transform of $\dfrac{d^2x}{dt^2} + 9x = \cos 2t, \ x(0) = 1, \ x(\pi/2) = -1$. **6**

b) Find the inverse Laplace Transform of $\dfrac{s^2 - 6}{s^3 + 4s^2 + 3s}$ **6**

OR

c) Solve equation **6**

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 4t + e^{3t}$$

when $x(0) = 1$ and $x'(0) = -1$.

d) Find inverse Laplace Transform of $\dfrac{1}{(s-2)(s+2)^2}$ by convolution theorem. **6**

5. Attempt **any six.**

a) Prove that **2**
$$a + b = a + c \Rightarrow b = c \ \forall \ a, b, c \in R$$

b) State law of trichotomy. **2**

c) Define intersection of two fuzzy set. **2**

d) Define union of crisp set. **2**

e) Find : $L\left[3t^2 - 2 \cdot e^t\right]$ **2**

f) If $L[f(t)] = F(s)$ then prove that $L\left[e^{at} f(t)\right] = F(s-a)$. **2**

g) Find Inverse Laplace transform of $\dfrac{s^2 - 3s + 4}{s^3}$. **2**

h) State the convolution theorem. **2**

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