B.Sc.-II (New CBCS Pattern) Semester - III

USMT-05 - Mathematics-I Paper-V : Real Analysis

P. Pages : 2 **GUG/S/23/11612**

* 1 7 4 3 *

Time : Three Hours Max. Marks : 60

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that a sequence can have at most one limit. **6**

b) Find the limit of the sequence $\langle S_n \rangle$ where **6**

$$S_n = \frac{1}{n^2+1} + \frac{1}{n^2+2} + ... + \frac{1}{n^2+n}$$

OR

c) Define a Cauchy sequence. Prove that every convergent sequence of real numbers is a Cauchy sequence. **6**

d) Show that the sequence $\langle S_n \rangle$ defined by $S_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ does not converge. **6**

UNIT – II

2. a) Prove that a series $\sum x_n$ of non-negative terms converges if and only if the sequence $\langle S_n \rangle$ of partial sums is bounded. **6**

b) Prove that a Geometric series $\sum_{n=1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for $0 < x < 1$ & diverges for $x \geq 1$. **6**

OR

c) Discuss the convergence of the series $\sum \frac{1}{n^2} \left(\frac{n-1}{n-2} \right)^n$. **6**

d) Test the convergence of the series $\sum \frac{n^3+a}{n^2+a}$ by D'Alembert's ratio test. **6**

UNIT – III

3. a) Show that $d : R \times R \to R$ defined by **6**

$$d(x, y) = \left| x^2 - y^2 \right| \ \forall x, y \in R$$

is a pseudo metric on R and is not a metric on R.

b) If p is a limit point of set A, then prove that every neighbourhood of p contains infinitely many points of A. **6**

OR

c) Let $\{A_\alpha\}$ be a finite or infinite collection of sets A_α then prove that $\left[\bigcup_\alpha A_\alpha\right]^C = \bigcap_\alpha {A_\alpha}^C$. **6**

d) Define a Cauchy sequence. Prove that every convergent sequence in a metric space is a Cauchy sequence. **6**

UNIT – IV

4. a) Show that any constant function defined on a bounded closed interval is integrable. **6**

b) If a function $f(x) = x$ on [0, 2] then show that f is integrable in Riemann sense over [0, 2] & $\int_0^2 f(x)\,dx = 2$. **6**

OR

c) Show that every continuous function is integrable. **6**

d) If f is monotonic in [a, b] then prove that f is integrable on [a, b]. **6**

5. Solve **any six.**

a) Evaluate $\lim_{n\to\infty} S_n$ if $S_n = \dfrac{2n^2 - n}{n+7}$ **2**

b) If $\lim x_n = x$ & $\lim y_n = y \neq 0$ then show that $\lim\left(\dfrac{x_n}{y_n}\right) = \dfrac{x}{y}$. **2**

c) Test the convergence of series $\sum x_n$ for $x_n = \dfrac{1}{x^2+2}$. **2**

d) Define the alternating series. **2**

e) Define metric. **2**

f) Define limit point of a set. **2**

g) For any partition P show that $L(P,f) \le U(P,f)$ **2**

h) Prove that $\int_0^1 e^{x^2}\,dx > 0$. **2**

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