B.Sc. (CBCS Pattern) Sem-III

# USMT-05 - Mathematics-I : Real Analysis

P. Pages : 2 **GUG/W/22/11612**

Time : Three Hours Max. Marks : 60

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Notes : 1. Solve all five questions.
2. Each questions carries equal marks.

## UNIT – I

**1.** a) Prove that a convergent sequence of a real numbers is bounded. **6**

b) Find the limit of the sequence $\langle s_n \rangle$ where, **6**

$$s_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \text{------} + \frac{1}{(n+n)^2}$$

**OR**

c) Let $\langle s_n \rangle$ be a sequence such that $s_n > 0 \ \forall n \in N$. Then prove that **6**

$$\lim_{n\to\infty} (s_n)^{1/n} = \lim_{n\to\infty} \frac{s_n}{s_{n-1}}$$

provided the limit on the right side exist.

d) Prove that a monotone sequence of real numbers is convergent if and only if it is bounded. **6**

## UNIT – II

**2.** a) Show that A geometric series $\sum_{n=1}^{\infty} x^{n-1}$ converges to $\frac{1}{1-x}$ for 0 < x < 1 and diverges for $x \geq 1$ **6**

b) Prove that a series $\sum x_n$ of non – negative terms converges iff the sequence $\langle s_n \rangle$ of partial sums is bounded. **6**

**OR**

c) Let $x_n \geq 0, y_n \geq 0 \forall n \in N$ and let $\exists$ $M \in N$ such that $x_n \leq k y_n \forall n \geq m, k > 0$. Then prove that **6**

i) $\sum y_n$ converges $\Rightarrow \sum x_n$ converges.

& ii) $\sum x_n$ diverges $\Rightarrow \sum y_n$ diverges

d) Test the convergence of the following series $\sum \frac{n^3 + a}{2^n + a}$ **6**

## UNIT – III

**3.** a) Show that the function d: $R \times R \to R$ defined by $d(x, y) = \left|x^2 - y^2\right| \forall x, y \in R$ is a pseudo metric on R & is not a metric on R. **6**

b) If p is a limit point of set A then prove that every neighbourhood of p contains infinitely many points of A. **6**

**OR**

c) Prove that for any finite collection $A_1, ---A_n$ of open sets $\bigcap_{i=1}^{n} A_i$ is open **6**

d) Prove that $\{s_n\}$ is a Cauchy sequence of real numbers if and only if $\{s_n\}$ is convergent in R. **6**

**UNIT – IV**

**4.** a) If f is a bounded and integrable function over $[a,b]$ and M, m are the bounds of f over $[a,b]$, then prove that **6**

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

b) Let the function f be defined as **6**

$$f(x) = \begin{cases} 1 & x \text{ is rational} \\ -1 & x \text{ is irrational} \end{cases}$$

show that f is not R – integrable over $[0,1]$

**OR**

c) Prove that a bounded function f defined on $[a,b]$ is integrable on $[a,b]$ iff for any $\in > 0$ there exists a $\delta > 0$ such that for every partition p of $[a,b]$ with $\mu(p) < \delta$, $U(p,f) - L(p,f) < \in$ **6**

d) Let the function f be integrable on $[a,b]$ and let $\alpha$ be any real constant. Then prove that $\alpha f \in R[a,b]$ and $\int_a^b (\alpha f)(x)dx = \alpha \int_a^b f(x)dx$ **6**

**5.** Solve **any six**.

a) Show that the sequence $\langle 1,0,1,0 ---- \rangle$ is not convergent. **2**

b) Evaluate $\lim_{n\to\infty} s_n$ for the sequence $s_n = \frac{2n^2 - n}{n+7}$ **2**

c) Test the convergence of the series $\sum_{n=1}^{n} x_n$ where $x_n = \cos\frac{\pi}{n}$ **2**

d) Test the convergence of series by using p – series test. **2**

$$\sum \frac{1}{n^3} = 1 + \frac{1}{2^3} + \frac{1}{3^3} + ----$$

e) Define closed set. **2**

f) Write definition of Cauchy sequence. **2**

g) For any partition p, prove that $L(p,f) \le U(p,f)$ **2**

h) Define Darboux's upper and lower sums. **2**

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