B.Sc. (Part-I) (CBCS Pattern) Sem-I

USMT-02 - Mathematics-II : Differential Calculus and Trigonometry

P. Pages : 2 **GUG/W/22/11557**

Time : Three Hours * 3 7 3 4 * Max. Marks : 60

Notes :
1. Solve all the **five** questions.
2. Question 1 to 4 have an alternative. Solve each question in full or its alternative in full.
3. All questions carry equal marks.

UNIT – I

1. a) By using $\in - \delta$ definition of limit of function prove that $\lim_{(x,y)\to(1,1)} (x^2 + 2y) = 3$. **6**

b) Let real valued functions f and g be continuous in an open set $D \subseteq R^2$. Then prove that $f + g$ is continuous in D. **6**

OR

c) If $u(x, y) = \frac{x^2 + y^2}{x + y}$ show that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$. **6**

d) If $H = f(y - z, z - x, x - y)$ then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$. **6**

UNIT – II

2. a) If $z = f(x, y)$ be a homogeneous function of x, y of degree n then prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$ **6**

b) If $xu = yz, yv = xz$ and $zw = xy$, then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. **6**

OR

c) If $u = \frac{x + y}{1 - xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u, v)}{\partial(x, y)}$, if $xy \neq 1$ state whether u and v are functionally related, If so find relationship. **6**

d) Find the maximum and minimum values of $x^3 + y^3 - 3axy$. **6**

UNIT – III

3. a) Find the tangent at the origin to the curve $x^3 + y^3 - 3axy = 0$. **6**

b) Find the radius of curvature at the point (x, y) on the curve $y^2 = 4ax$. **6**

OR

c) Find the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1$. **6**

d) Trace the curve $x^3 + y^3 = 3axy$. **6**

UNIT – IV

4. a) Prove that, if n is a positive and negative then $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. **6**

b) If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$ then show that $x^m \cdot y^n + \frac{1}{x^m \cdot y^n} = 2\cos(m\theta + n\phi)$ **6**

OR

c) Find all the values of $(32)^{1/5}$. **6**

d) If $\sin(\theta + i\phi) = \cos\alpha + i\sin\alpha$. Then prove that $\cos^2\theta = \pm\sin\alpha$. **6**

5. Solve **any six.**

a) Define Continuity of function $f(x, y)$. **2**

b) Using algebra of limits evaluate $\lim_{(x,y)\to(1,2)} (x^2 + xy - 2x - y)$. **2**

c) If $u = x^2, v = y^2$ find $\frac{\partial(u, v)}{\partial(x, y)}$. **2**

d) Find the critical point of function $f(x, y) = x^3 - 3xy^2$. **2**

e) Find the tangent at the point (1, 3) to the curve $y = x^3 + 2$. **2**

f) Find the radius of curvature at any point on the curve $S = C\tan\psi$. **2**

g) Express the complex number in polar form $1 + i$. **2**

h) Prove that $\sin(iz) = i\sinh z$ **2**

****