B.Sc. (CBCS Pattern) Semester - I

USMT-02 - Mathematics-II (Differential Calculus and Trigonometry)

P. Pages : 2 **GUG/S/23/11557**

Time : Three Hours *1688* Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) If limit of a function $f(x, y)$ as $(x, y) \to (x_0, y_0)$ exists, then prove that it is unique. **6**

b) Using $\in - \delta$ definition of a limit of a function, prove that **6**

$$\lim_{(x,y)\to(4,-1)} (3x - 2y) = 14$$

OR

c) If $u = 3(ax + by + cz)^2 - (x^2 + y^2 + z^2)$ and $a^2 + b^2 + c^2 = 1$, **6**

show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

d) If $z = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, then show that **6**

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

UNIT – II

2. a) If $u = f(x, y)$ is a homogeneous differentiable function of degree n in x, y then **6**

prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

b) Verify Euler's theorem on homogeneous function for $3x^2yz + 5xy^2z + 4z^4$ **6**

OR

c) If $x + y + z = u, y + z = uv, z = uvw$, prove that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2v$. **6**

d) Expand $x^3 + y^3 - 3xy$ in power of $x - 2$ and $y - 3$ i.e. at the point (2, 3). **6**

UNIT – III

3. a) For the curve $x^{m+n} = a^{m-n} y^{2n}$, prove that the m^{th} power of the sub tangent varies as the n^{th} power of the subnormal. **6**

b) Find the radius of curvature at the point (x, y) on the curve $y^2 = 4ax$. **6**

OR

c) Find the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 = 1$. **6**

d) Find the asymptotes of $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$. **6**

UNIT – IV

4. a) If n is positive and negative integer then prove that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ where $\theta \in R$. **6**

b) If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, prove that $\alpha^n + \beta^n = 2^{n+1}\cos\frac{n\pi}{3}$. **6**

OR

c) Separate into real and imaginary parts. **6**
i) $\tan(x + iy)$ ii) $\sec(x + iy)$

d) If $\sin(\alpha + i\beta) = x + iy$, prove that $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\sinh^2\beta} = 1$ and $\frac{x^2}{\sin^2\alpha} - \frac{y^2}{\cos^2\alpha} = 1$. **6**

5. Solve **any six.**

a) Show that $\lim_{(x,y)\to(3,1)} \frac{\tan^{-1}(xy-3)}{\sin^{-1}(4xy-12)} = \frac{1}{4}$. **2**

b) If $u = \frac{e^{x+y+z}}{e^x + e^y + e^z}$, show that $u_x + u_y + u_z = 2u$ **2**

c) Define – Homogeneous function. **2**

d) For the following mapping, find Jacobian determinant of the mapping and inverse mapping $u = 2x - y, v = x + 4y$. **2**

e) Find the tangent and normal at (1, 3) to the curve $y = x^3 + 2$. **2**

f) Find ρ at the point (s, ψ) for the curve (i) $s = c\tan\psi$, (ii) $s = 4a\sin\psi$. **2**

g) Show that $\frac{\left[\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]^{11/2}}{\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]^{1/2}} = -1$. **2**

h) Express 1 – i in polar co-ordinate. **2**

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