B.Sc.-I (CBCS Pattern) Semester - I

USMT-01 - Mathematics Paper-I (Differential and Integral Calculus)

P. Pages : 3 **GUG/S/23/11556**

Time : Three Hours * 1 6 8 7 * Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Show that **6**

$$\lim_{x\to1}\frac{2x^3-x^2-8x+7}{x-1}=-4$$, by using $\in-\delta$ technique.

b) If f and g are two continuous at $x=a$. Then prove that $f-g,\ f\cdot g$ and $\frac{f}{g},\ g(a)\neq0$ are continuous at $x=a$. **6**

OR

c) Find the nth derivatives of $\cos^6 x$. **6**

d) If $y=e^{a\sin^{-1}x}$. Then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$. **6**

UNIT – II

2. a) If $f(x)$ and $g(x)$ are continuous real functions on [a, b] which are differentiable in (a, b), then prove that there is a point $c\in(a,b)$ such that **6**

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)},$$

where $g(a)\neq g(b)$ and $f'(x),\ g'(x)$ are not simultaneously zero.

b) Verify Rolle's theorem for the function **6**

$f(x)=x^2+x-6$ in $[-3,2]$.

OR

c) Show that **6**

$$\log(x+h)=\log h+\frac{x}{h}-\frac{x^2}{2h^2}+\frac{x^3}{3h^3}-........$$

d) Obtained the Maclaurin's series for the function $f(x)=\log(1+x)$. **6**

UNIT – III

3. a) Prove that **6**

$$\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$$

b) Show that **6**

$$\int_0^1 x^m (\log x)^n \,dx = \frac{(-1)^n\, n!}{(m+1)^{n+1}}$$

Where n is a positive integer and $m > -1$.

OR

c) Prove that **6**

$$\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x} = -\frac{e}{2}$$

d) Evaluate **6**

$$\lim_{x\to 1}\left[\frac{1}{\log x} - \frac{x}{x-1}\right]$$

UNIT – IV

4. a) Let $f(x,y)$ and $g(x,y)$ be the continuous functions on the region D, then prove that **6**

$$\iint_D [f(x,y) + g(x,y)]\,dA = \iint_D f(x,y)\,dA + \iint_D g(x,y)\,dA$$

b) Evaluate $\int\int xy(x+y)\,dx\,dy$, where R is the region bounded by the curves $y = x^2$ and $y = x$. **6**

OR

c) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. **6**

d) Evaluate $\iint_D \frac{x^2}{\sqrt{x^2+y^2}}\,dx\,dy$ by changing to polar coordinates, where D is the region designed by $0 \le x \le y,\ 0 \le x \le a$. **6**

5. Solve **any six.**

a) Evaluate by limit theorems **2**

$$\lim_{x\to 3}(2x^3 - 3x^2 + 7x - 11)$$

b) If $y = A\sin mx + B\cos mx$. **2**

Prove that $y_2 + m^2 y = 0$.

c) State Taylor's theorem for the function $f(x)$ about the point x_0. **2**

d) State the Lagrange's mean value theorem. **2**

e) Evaluate: **2**

$$\int_0^{\infty} x^3 e^{-2x}\, dx$$

f) Evaluate: **2**

$$\int_0^{\pi/2} \sin^2\theta \cos^4\theta \, d\theta$$

g) If $f(x, y) \geq 0$. Then prove that **2**

$$\iint_R f(x, y)\, dA \geq 0 \text{ on R.}$$

h) Evaluate the double integral **2**

$$\int_0^1 \int_0^2 (x + 2)\, dx\, dy$$

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