B.Sc.- I (CBCS Pattern) Sem-I

USMT-01 - Mathematics Paper-I : Differential And Integral Calculus

P. Pages : 2 | GUG/W/22/11556

Time : Three Hours | * 2 2 2 1 * | Max. Marks : 60

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Let f(x) and g(x) be the two functions such that **6**

$\lim\limits_{x \to x_0} f(x) = \ell$ and $\lim\limits_{x \to x_0} g(x) = \ell m$.

Then by $\in - f$ technique, prove that

$\lim\limits_{x \to x_0} [f(x) \cdot g(x)] = \ell . m$

b) If $f(x) = \dfrac{e^{1/x}}{1 + e^{1/x}}$, $x \neq 0$ **6**

= 0, x = 0, show that f(x) has simple discontinuity at x = 0.

OR

c) If $y = A\, e^{-mt} \cos(pt + q)$. Then show that **6**

$\dfrac{d^2y}{dt^2} + 2m\dfrac{dy}{dt} + n^2 y = 0$, where $n^2 = p^2 + m^2$

d) If $y = \left(x + \sqrt{1 + x^2}\right)^m$, then prove that $\left(1 + x^2\right) y_{n+2} + (2n+1)x\ y_{n+1} + \left(n^2 - m^2\right) y_n = 0$ **6**

UNIT – II

2. a) If a function f(x) defined on [a,b] such that f(x) is continuous in [a,b] and derivable in (a,b), then prove that there exists at least one value $c \in (a, b)$ such that **6**

f ' (c) (b - a) = f(b) – f(a).

b) Verify Cauchy mean value theorem for the functions **6**

$f(x) = x^2,\ g(x) = x^3$ in $[1,3]$

OR

c) Obtain Taylor's series for the function $f(x) = (1 - x)^{5/2}$ with Lagrange's form of remainder up to three terms in the interval [0,1]. **6**

d) Expand $2x^3 + 7x^2 + x - 1$ in powers of (x – 2). **6**

UNIT – III

3. a) If $\dfrac{\pi}{\sin p\pi} = \int_0^\infty \dfrac{x^{p-1}}{1+x} dx$, for 0 < p < 1 then show that **6**

$\overline{|P}\ \overline{|1 - P} = \dfrac{\pi}{\sin p\pi}$ and hence

evaluate $\int_0^\infty \dfrac{dy}{1 + y^4}$

b) Prove that **6**

$$\overline{|n+1} = n\overline{|n}$$

OR

c) Evaluate **6**

$$\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{\frac{1}{x}}$$

d) Prove that **6**

$$\lim_{x\to 0}\log_{\tan x}\tan 2x = 1$$

UNIT – IV

4. a) If the region D of the integration is the triangle bounded by y = 0, y = x and x= 1, then show that **6**

$$\iint_D \sqrt{4x^2-y^2}\,dxdy = \frac{1}{3}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)$$

b) Evaluate $\iint_D \frac{dxdy}{x^4+y^2}$, where D is the region $x \ge 1,\ y \ge x^2$ **6**

OR

c) Evaluate $\int_0^1\int_x^{\sqrt{2-x^2}} \frac{x\ dy\ dx}{\sqrt{x^2+y^2}}$, **6**

by changing the order of integration.

d) Evaluate $\int_0^\infty\int_0^\infty e^{-\left(x^2+y^2\right)}dx\ dy$ by changing to polar coordinates. **6**

5. a) Show that $\lim_{x\to 0} f(x)$ does not exists if **2**

$$f(x) = 2x-1,\quad x<0$$
$$= 2x+1,\quad x>0$$

b) If y = sin (1-x). Find y_8. **2**

c) State the Rolle's theorem. **2**

d) Verify Lagrange's mean value theorem for f(x) = log x in [1,e] **2**

e) Evaluate $\beta\left(\frac{3}{2}, {}^5\!/_2\right)$ **2**

f) Prove that $\overline{|\pi} = 1$ **2**

g) If f(x,y) is the function defined on the region D and C is constant then prove that **2**

$$\iint_D C\ f(x,y)d\ A = C\iint_D f(x,y)dA$$

h) Evaluate $\int_0^1\int_1^3 xy^2dy\ dx$ **2**

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