B.Sc. (CBCS Pattern) Sem-I

USMT-01 - Mathematics -I : Differential and Integral Calculus

P. Pages : 3 | GUG/W/22/11556

Time : Three Hours | Max. Marks : 60

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Notes : 1. Solve all the five questions.
2. All question carry equal marks.

UNIT - I

1. a) If $\lim_{x \to x_0} f(x)$ exists, then prove that it is unique. **6**

b) If $y^{1/m} + y^{-1/m} = 2x$, prove that **6**

$\left(x^2 - 1\right) y_{n+2} + (2n+1) x y_{n+1} + \left(n^2 - m^2\right) y_n = 0$

OR

c) If f, g are continuous functions at $x = x_0$ then prov that $f + g$, $f \cdot g$, are continuous at $x = x_0$ and if $g(x_0) \neq 0$, then prove that $\frac{f}{g}$ is also continuous at x_0. **6**

d) If $\cos^{-1}(y/b) = \log(x/n)^n$, prove that **6**

$x^2 y_{n+2} + (2n+1) x y_{n+1} + 2n^2 y_n = 0$

UNIT – II

2. a) If f and g are continuous real function on [a, b] which are differentiable in (a, b), then prove that there is a point $c \in (a, b)$ such that $\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)'}$ where $g(a) \neq g(b)$ and $f'(x), g'(x)$ are not Simultaneously zero. **6**

b) Obtain Maclaurains series for $f(x) = \log(1 + x)$. **6**

OR

c) If a real function f defined on [a, b] is **6**

i) Continuous on [a, b] and

ii) Differentiable on (a, b), then prove that there is at least one point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

d) Show that $e^{x \cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} + ----$ **6**

UNIT – III

3. a) Show that $\beta(m,n)=\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$ **6**

b) Evaluate $\lim_{n\to\infty}\left(\frac{a^{1/x}+b^{1/x}+c^{1/x}}{3}\right)^x$ **6**

OR

c) Prove that $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}\int_0^{\pi/2}\sqrt{\sin\theta\, d\theta}=\pi$. **6**

d) Evaluate $\lim_{n\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$ **6**

UNIT – IV

4. a) Prove that **6**

i) $\iint_D cf(x,y)dA = c\iint_D f(x,y)dA$, c is constant.

ii) $\iint_D\left[f(x,y)+g(x,y)\right]dA=\iint_D f(x,y)dA+\iint_D g(x,y)dA$

b) Evaluate by changing to polar coordinates. **6**

$\int_0^\infty\int_0^\infty e^{-\left(x^2+y^2\right)}dx\cdot dy.$

OR

c) Evaluate $\iint_D x^2\,dxdy$, where D is the region in the first quadrant bounded by the hyperbola xy = 16 and the lines y = x, y = 0 and x = 8. **6**

d) Evaluate by changing the order of integration. **6**

$\int_0^2\int_{-1}^{1-y}x^{1/3}y^{-1/2}(1-x-y)^{1/2}dx\cdot dy$

5. Solve **any six**

a) Evaluate $\lim_{x\to 3}\left(2x^3-3x^2+7x-11\right)$ **2**

b) If $y^3+3ax^2+x^3=0$, then show that $y^5y_2+2a^2x^2=0$ **2**

c) Obtain Maclaurin's series for $f(x) = \sin x$ **2**

d) Define Taylors polynomial for f at the point C. **2**

e) Evaluate $\lim_{x \to \infty} \frac{\log\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$ **2**

f) Evaluate $\int_0^\infty x^m e^{-ax^n} dx$, where m, n, a are positive constant. **2**

g) Evaluate $\int_0^1 \int_1^3 xy^2 dydx$ **2**

h) Evaluate $\iint r \sin\theta \, dr \, d\theta$ over the area of the cardioid $r = a(1-\cos\theta)$ above the initial line.

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