

P. Pages : 2

Time : Three Hours

**GUG/W/22/11180**

Max. Marks : 80

Note : All questions are compulsory and carry equal marks.

Either:

1. a) Find the values of c_1 and c_2 such that the function **8**
 $f(z) = x^2 + c_1y^2 - 2xy + i(c_2x^2 - y^2 + 2xy)$
 is analytic. Also find $F'(z)$

- b) Evaluate : $\int_C \frac{2}{(z^2 - 3z + 2)} dz$ **8**
 by using Cauchy's integral formula where C is circle $|z - 2| = \frac{1}{2}$

OR

- e) Prove that If a function $F(z)$ is analytic and its derivative $F'(z)$ continuous at all points **8**
 inside and an a simple closed curve C, then $\int_C f(z) dz = 0$

- f) Find the modulus and argument of the following complex numbers **8**
 i) $\frac{1+2i}{1-(1-i)^2}$ ii) $\frac{(1+i)^2}{1-i}$

Either:

2. a) How one can find the residue- **8**
 i) At simple pole ii) At pole of order n
- b) Determine the poles and the residue at each pole of the function **8**
 i) $F(z) = \frac{z^2}{(z-1)^2(z+2)}$ ii) $F(z) = \text{crt } z$

OR

- e) Determine the poles of the function $z f(z) = \frac{1}{z^4 + 1}$ **8**
- f) Define singularity of a function. Find the singularities of the functions **8**
 i) $f(z) = \sin \frac{1}{z}$ ii) $g(z) = \frac{e^{1/z}}{z^2}$

Either:

3. a) Using Bisection methods find the root of $x^3 - 5x + 3 = 0$ correct upto 4 decimal places **8**
- b) Find the root of equation $\cos x = 3x - 1$ using iteration method correct upto 3 decimal places **8**

OR

- e) Obtain the secant general formula for finding the root of the equation. **8**
- f) Define finite difference. Explain the different types of finite differences. **8**

Either:

4. a) Deduce the formula for linear least squares. **8**
- b) Obtain the general formula for trapezoidal rule and show it graphically. **8**

OR

- e) Explain Simpson's $\frac{1}{3}$ rd rule and obtain the formula for it **8**

- f) Evaluate $\int_1^2 e^{-x/2} dx$ using Simpson's $1/3$ rd rule **8**

5. Attempt all of the following

- a) Explain Bisection method. **4**
- b) Explain branch points. **4**
- c) Define complex number and show that the sum and product of complex number and its conjugate complex are both real. **4**
- d) What are divided differences? **4**
