M.Sc.(Physics) (CBCS Pattern) Semester - I

PSCPHYT01 - Core Paper-I : Mathematical Physics

P. Pages : 2 | GUG/S/23/11179

Time : Three Hours | Max. Marks : 80

Either:

1. a) What are curvilinear coordinates. Obtain an expression for divergence of a vector field in Curvilinear coordinate system. **8**

b) Find Fourier sine transform of **8**

$$F(x)=\frac{e-ax}{x}$$

OR

e) What are Scalar and Vector fields? **8**

Define-

i) Time derivatives of vector field

ii) Gradient of a Scalar function

iii) Divergence and curl of a vector.

f) A vector field defined by **8**

$\vec{A}=\hat{r}r^n$, where $r=\left(x^2+y^2+z^2\right)^{1/2}$ and $\hat{r}$ is the unit vector from origin to the point (x,y,z). Then find divergence and curl of a vector $\vec{A}$.

Either:

2. a) State and Prove contraction theorem of Tensor. **8**

b) Prove the following. **8**

i) grad (fg) = $f\times\text{curl}\ \bar{g}+\bar{g}\times\text{curl}\,f+f\ \nabla\bar{g}+\bar{g}\,\nabla f$

ii) grad (divf) = curl (curl f) $+\nabla^{\wedge}2f$

OR

e) Explain contravariant, covariant and mixed tensor of rank two. Show that mixed tensor of rank two is not symmetric in coordinate system. **8**

f) Define a metric or fundamental tensor. Determine the components of the fundamental tensor in cylindrical coordinates. **8**

Either:

3. a) State and prove Cayley - Hamilton theorem. **8**

b) Find the A^{-1} of the matrix by using C - H theorem. **8**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

e) If 'A' is a Unitary matrix show that A^T is also Unitary. **8**

f) Find a matrix P, which is diagonalizes the matrix. **8**

$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ Verify $P^{-1}AP = D$

Where 'D' is the diagonal matrix.

Either:

4. a) Prove that, For Bessel's functions $J_n(x)$. **8**

$$J_{-n}(x) = (-1)^n J_n(x)$$

b) Prove that, Recurrence formula. **8**

$$xJ_n' = nJ_n - xJ_{n+1}$$

OR

e) Express $F(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomial. **8**

f) Prove that, $J_n(x)$ is the coefficient of $'Z^n'$ in the expansion of $e^{x/2}(z-1/2)$ **8**

5. Attempt all the following:

a) Find the Fourier Sine Transform of $F(x) = e^{-ax}$, for $0 < x < \infty$ **4**

b) Define Inner product space and its properties. **4**

c) Prove that, $H'_{2n}(0) = 0$ **4**

d) Define divergence of a vector and give its physical meaning. **4**

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