M.Sc. (Physics) (CBCS Pattern) Sem-I

PSCPHYT01 - Paper-I - Core-I : Mathematical Physics

P. Pages : 2 **GUG/W/22/11179**

Time : Three Hours Max. Marks : 80

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Either:

1. a) What are curvilinear coordinates? Obtain an expression for divergence of a vector field in curvilinear coordinate system. **8**

b) Find Curl and Divergence of $\bar{v}$ **8**

$$\bar{v} = \frac{x\hat{i} + y\hat{j} + zk}{\sqrt{x^2 + y^2 + z^2}}$$

OR

e) What are scalar and vector fields? Define. **8**

i) Time derivatives of vector field

ii) Gradient of a scalar function

iii) Divergence and curl of a vector.

f) A vector field defined by **8**

$\bar{A} = \hat{r}r$, where $r = \left(x^2 + y^2 + z^2\right)^{1/2}$ and $\hat{r}$ is the unit vector from origin to the point (x, y, z). Then find divergence and curl of a vector $\vec{A}$.

Either:

2. a) State and prove contraction theorem of tensor. **8**

b) Prove the following. **8**

i) $\text{grad}\left(\bar{f} \cdot \bar{g}\right) = \bar{f} \times \text{curl}\,\bar{g} + \bar{g} \times \text{curl}\,\bar{f} + \bar{f} \cdot \bar{\nabla} g + \bar{g} \cdot \nabla \bar{f}$

ii) $\text{grad}\left(\text{div} \cdot \bar{f}\right) = \text{curl}\left(\text{curl}\,\bar{f}\right) + \nabla^2 \bar{f}$

OR

e) Explain contravariant, covariant and mixed tensor of rank two. Show that mixed tensor of rank two is not symmetric is any coordinate system. **8**

f) Define a metric or fundamental tensor. Determine the components of the fundamental tensor in cylindrical coordinates. **8**

Either:

3. a) State and prove Cayley – Hamilton theorem **8**

b) Verify Cayley – Hamilton theorem for the matrix **8**

$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and hence find A^{-1}

OR

e) If ‘A’ is a unitary matrix. Show that A^T is also Unitary. **8**

f) Find a matrix ‘P’ which is diagonalizes the matrix. **8**

$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$, verify $P^{-1}AP = D$, where ‘D’ is the diagonal matrix.

Either:

4. a) Prove that for Bessels function $J_n(x)$ **8**

$J_{-n}(x) = (-1)^n \cdot J_n(x)$

b) Prove that recurrence formula. **8**

$xJ_{n'} = nJ_n - xJ_{n+1}$

OR

e) Prove that $\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{2n+1}$ **8**

f) Prove that $J_n(x)$ is the coefficient of $'Z^n'$ in the expansion of $e^{\frac{x}{2}\left(z - \frac{1}{2}\right)}$ **8**

5. Attempt all the following.

a) Find the Fourier sine transform of $F(x) = e^{-ax}, \text{for}, 0 < x < \infty$ **4**

b) Define Inner product space and its properties. **4**

c) Find the Inverse Laplace transform of $\frac{s^2 - a^2}{\left(s^2 + a^2\right)^2}$ **4**

d) Prove that, $H'_{2n}(0) = 0$ **4**

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