M.Sc. (Physics) (CBCS Pattern) Sem-I

PSCPHYT01 - Core-I - Paper-I : Mathematical Physics

P. Pages : 3 **GUG/W/22/11179**

Time : Three Hours Max. Marks : 80

Notes : 1. All questions are compulsory.

Either:

1. a) If a vector field is given by: **8**

$\vec{F} = \left(x^2 - y^2 + x\right)\hat{i} - (2xy + y)\hat{j}$. Is this field irrotational? If so find its scalar potential.

b) Find sine and cosine transform of: **8**

$$f(x) = \begin{cases} 1 + \dfrac{x}{a} & -a < x < 0 \\ 1 - \dfrac{x}{a} & 0 < x < 0 \\ 1 + \dfrac{x}{a} & \text{otherwise} \end{cases}$$

OR

e) Find the Fourier series for: **8**

$$f(x) = \begin{cases} 0 - \pi < x < 0 \\ x \quad 0 < x < \pi \end{cases}$$

f) Define curl of a vector? If V is a vector field then find curl of V in terms of curvilinear Coordinates. **8**

Either:

2. a) If there be an entity represented by multi suffix set a_{ij} relatively to any given system of rectangular axes and if $a_{ij}\, b_i$ is a vector, where b_i is any arbitrary vector whatsoever then a_{ij} is a tensor of order two. **8**

b) Define Christoffel symbols of first and second kind and prove that. **8**

$$\frac{\partial g^{p \cdot q}}{\partial x^m} = -g^{p \cdot n} \left\{ {q \atop m \cdot n} \right\} - g^{p \cdot n} \left\{ {p \atop m \cdot n} \right\}$$

OR

e) What do you mean by symmetric and antisymmetric tensor? Show that and any second order tensor can be expressed as the Sum of symmetric and skew symmetric tensors. **8**

f) What are metric tensors? Obtain the components of metric tensor in three dimensional space in terms of spherical polar coordinates. **8**

Either:

3. a) Find eigen value of A^3 if **8**

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Satisfy the Cayley-Hamilton theorem and find A^{-1} of the matrix. **8**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

OR

e) Diagonalise **8**

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

f) The matrix A is defined as **8**

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

Find the eigen values of $3A^3 + 5A^2 - 6A + 2I$.

Either:

4. a) Solve the differential equations. **8**

i) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$

ii) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cdot \cosh 2x$

b) Find the power series solution of **8**

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

OR

e) Express $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials. **8**

f) Prove that for Bessel's function $J_n(x), J_{(-n)}(x) = (-1)^n J_n(x)$. **8**

5. Answer the following.

a) Find the Fourier expansion of time period of 2π. Where $f(x) = x^2, -\pi < x < \pi$. **4**

b) What is contravariant and covariant tensor? **4**

c) Show that the matrix. **4**

$A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is a unitary matrix if $\alpha^2 + \gamma^2 + \beta^2 + \delta^2 = 1$.

d) Solve differential equation: **4**

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5e^{3x}$$

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