M.Sc. (Mathematics) (CBCS Pattern) Sem-I

PSCMTHT05 - Paper-V : Numerical Analysis

P. Pages : 2 **GUG/W/22/11170**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all five questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Apply Newton's method to the following function **10**

$$f(x) = \sqrt{x}, x \geq 0$$
$$= -\sqrt{-x}, x < 0$$

With root $\alpha = 0$. What is the behaviour of the iterates? Do they coverage and if so, at what values.

b) Assume what $f(x), f'(x), f''(x)$ are continuous for all values of x in some interval containing α and assume $f'(\alpha) \neq 0$. Then prove that if the initial guess x_0 and x_1 are chosen sufficiently close to α, the iterates x_n will converges to α. The order of convergence will be $P = \frac{1+\sqrt{5}}{2} \cong 1.62$ **10**

OR

c) Assume $f(x), f'(x)$ and $f''(n)$ continuous for all x in some neighborhood of α and assume that $f(\alpha) = 0, f'(\alpha) \neq 0$ then prove that if x_0 is chosen sufficiently close to α the iterates $x_n, n \geq 0$ will converges to α. **10**

d) Consider $f(x) = (x-1)\left(x - \frac{1}{2}\right)$ with $\alpha = 1, \frac{1}{2} < a < 1, b = 2$. Show that for this example the Regula Falsi method is linear with a speed of convergence 2/3 **10**

UNIT – II

2. a) Let $x_0, x_1, ---------x_n$ be distinct real number and Let f be given real valued function with n + 1 continuous derivative on the interval $I_1 = H\{t, x_0, x_1 ----x_n\}$ with t some given real number. Then prove that there exist $\xi \in I$, with **10**

$$f(t) - \sum_{j=0}^{n} f(x_j) \ell_j(t) = \frac{(t-x_0)----(t-x_n) f^{n+1}(\xi)}{(n+1)1}$$

b) For any two function f and g and any two constant α, β prove that **10**

$$\Delta^r [\alpha f(x) + \beta g(n)] = \alpha \Delta^r f(x) + \beta \Delta^r g(n)$$

OR

c) Find out Hermite inter polating polynomial for with **10**

$p(a) = f(a), p'(a) = f'(a)$

$p(b) = f(b), p'(b) = f'(b)$

d) For the basis functions $\ell_{j,n}(x)$ **10**

Prove that for any $n \geq 1 \sum_{j=0}^{n} \ell_{j,n}(x) = 1$

For all x

UNIT – III

3. a) Obtain the Chebyshev linear polynomial approximation to the function $f(x) = x^3$ on $[0,1]$ **10**

b) To obtain a minimax polynomial approximation $a_1^*(x)$ to e^x on the interval [-1, 1] **10**

OR

c) Apply economization twice to obtain the linear approximation in the following cases **10**

$f(x) = e^x, p_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x3}{3!}$ and $\|e^x - p_3(x)\| = 0.0516$

d) Using weight functions $w(x) = 1$ on the interval [-1, 1] construct first two term of the sequence of orthogonal polynomial by Gram Schmidt process. **10**

UNIT – IV

4. a) Obtain Peano Kernel formulation of the error for simple trapezoidal rule. **10**

b) Obtain the expression for the Bernoulli Polynomial $\beta_n(x), n = 1, 2, 3, 4$ **10**

OR

c) Show that when Simpson's rule is applied to $\int_0^{2\pi} \sin x dx$, there is a zero error, assuming no rounding error. Also show that Simpson's rule is exact for $\int_0^{2\pi} \cos^2 x dx$ **10**

d) Obtain Newton – cotes integration formula for n = 1. **10**

5. a) Let $g(x)$ be continuous on the interval $a \leq x \leq b$ and assume that $a \leq g(n) \leq b$ for every $a \leq x \leq b$ then x = g (x) has at least one solution in [a, b] **5**

b) Obtain the expression for $p_1(x)$ by Lagrange interpolation. **5**

c) For $f, g \in c[a, b]$ then prove that **5**

$\|f + g\|_2 \leq \|f\|_2 + \|g\|_2$

d) Obtain Simple trapezoidal Rule. **5**

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