M.Sc. I (Mathematics) (CBCS Pattern) Sem-I

PSCMTHT03 - Paper-III : Topology-I

P. Pages : 2

GUG/W/22/11168

Time : Three Hours

* 1 9 2 2 *

Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Prove that countable union of countable set is countable. **10**

b) Show that a set of all real numbers is uncountable. **10**

OR

c) Prove that every infinite set contains a denumerable subset. **10**

b) Prove that $2^{\text{No}} = C$. **10**

UNIT – II

2. a) Prove that f is closed iff its complement is open. **10**

b) Prove that Y* is a topology for X*. **10**

OR

c) Prov that $i(E) = \sim C(\sim E)$. **10**

d) Let E be a subset of a topological space (X, J) prove that **10**

i) c (E) = E U B (E)

ii) i (E) = E – b (E)

iii) b (E) = ϕ if and only if E is both open and closed. [Here c, i and b denote closure, interior and boundary operator respectively].

UNIT – III

3. a) Prove that a closed subset of a locally compact topological space is locally compact. **10**

b) If F is a continuous mapping of a topological space X into another topological space X* then prove that F maps every connected subset of X into a connected subset of X* **10**

OR

c) If C is connected subset of a topological space (X, J) which has separation X=A/B then prove that $C \subseteq A$ or $C \subseteq B$. **10**

d) If C is a connected subset of a topological space (x, τ) which is it self connected and (X-C) has a separation A/B, then prove that $A \cup C$ and $B \cup C$ both are connected. **10**

UNIT – IV

4. a) Prove that, compact subset E of a Hausdorff topological space is closed. **10**

b) Prove that T_1-space X is countably compact iff every countably open covering of X is reducible to a finite subcover. **10**

OR

c) Prove that Let (X, J) be a topological space prove that X is T_1-space iff every singleton set in X is closed. **10**

d) Prove that A topological space X is regular iff only point $x \in X$ and any open set G in X such that $x \in G$ there exist an open set G* in X such that $x \in G^*$ and $c(G^*) \subseteq G$. **10**

5. a) Show that the set of integers Z is denumerable. **5**

b) Prove that if F is a closed set then CF is an open set. **5**

c) Prove that closure of connected set is connected. **5**

d) Give an example of To space which is not T_1-space. **5**

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