M.Sc. I (Mathematics) (CBCS Pattern) Sem-I

PSCMTHT02 - Paper-II : Real Analysis-I

P. Pages : 2 **GUG/W/22/11167**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) Suppose $f_n \to f$ uniformly an a set E in a metric space. Let x be a limit point of E & suppose that **10**

$$\lim_{t\to x} f_n(t) = A_n,\ n = 1, 2, ...$$

Then prove that

$\{A_n\}$ converges & $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.

b) Prove that a convergent series of continuous function may have discontinuous sum. **10**

OR

c) Let α be a monotonically increasing on [a, b] let $fn\, E\, R(\alpha)$ on $[a,b]$ for n = 1, 2,….. & let $f_n \to f$ uniformly on [a, b] then show that $f \in R(\alpha)_1$ on $[a,b]$ & **10**

$$\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n\, d\alpha$$

d) State & prove the Stone-Weierstrass theorem. **10**

UNIT – II

2. a) Suppose that E be an open set in R^n and $f : E \to R^n$, $x \in E$ & **10**

$$\lim_{h\to 0} \frac{|f\ (x+h) - f\ (x) - Ah\ |}{|h|} = 0$$

holds for $A = A_1$ and $A = A_2$ where $A_1,\ A_2 \in L\left(R^n, R^m\right)$. Then prove that $A_1 = A_2$.

b) Suppose that f maps an open set $E \subset R^n$ into R^m and f is differentiable at a point $x \in E$. Then prove that the partial derivatives $\left(D_j\ f_i\right)(x)$ exist and **10**

$$f'(x)e_j = \sum_{i=1}^{m} \left(D_j f_j\right)(x) u_i,\ 1 \le j < n$$ where $\{e_1,\ e_2, ..., e_n\}$ and $\{u_1,\ u_2, ..., u_m\}$ are standard bases of R^n and R^m respectively.

OR

c) State and prove the inverse function theorem. **10**

d) Let f maps open set $E \subset R^n$ int o R^m. Then prove that $f \in C'(E)$ if and only if the partial derivative $D_i f_i$ exist & are continuous on E for $l \le i \le m,\ l \le j \le n$. **10**

UNIT – III

3. a) Prove that: **10**

The vector space $T_a\left(R^n\right)$ is isomorphic to the vector space D (a) of all derivations of $C^\infty(a)$ into R. This isomorphism is given by making each X_a correspond to the directional derivative X_a^* in the direction of X_a.

b) Let M be a Hausdorff space with a countable basis of open sets. If $V=\{(V_\beta, \psi_\beta)\}$ is a covering of M by $C^\infty -$ compatible coordinate neighbourhoods, then prove that there is a unique $C^\infty -$structure on M containing these coordinate neighbourhoods. **10**

OR

c) Define diffeomorphism between two C^∞ manifolds & show that by an example that C^∞ homomprhism may not be a diffeomorphism. **10**

d) Let ~ be an open equivalence relation on a topological space X then prove that $R=\{(x,y)/x \sim y\}$ is a closed subset of the space $X \times X$ if and only if the quotient space $X/\sim$ is Hausdorff. **10**

UNIT – IV

4. a) Prove that S^2 is a regular submanifold of R^3. **10**

b) If G_1 & G_2 are lie group then show that the direct product $G_1 \times G_2$ of these group with C^∞ structure of the Cartesian product of manifold is a Lie group. **10**

OR

c) Let $f: N \rightarrow M$ be an immersion then show that each $p \in N$ has a neighbourhood U such that F/U is an imbedding of U in M. **10**

d) Let a map $f:(1,\infty) \rightarrow R^2$ given by $f(t)=\left(\frac{\cos 2\pi t}{t}, \frac{\sin 2\pi t}{t}\right)$, then describe the image of f & prove that f is an immersion. **10**

5. a) If $f_n = n^2 x\left(1-x^2\right)^2$, $(0 \le x \le 1,\ n = 1,2,....)$ then show that the limit of the integral need not be equal to the integral of limit. **5**

b) Define projection in a vector space. State two properties of projection. **5**

c) Define a differentiable manifold. **5**

d) Define- **5**
- i) Imbedded submanifold
- ii) Regular submanifold
- iii) Lie group

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