M.Sc. I (Mathematics) (CBCS Pattern) Sem-I

PSCMTHT01 - Paper-I : Algebra-I

P. Pages : 2 | GUG/W/22/11166

Time : Three Hours | Max. Marks : 100

Notes : 1. Solve all the **five** questions.
2. All questions carry equal marks.

UNIT – I

1. a) State and prove the third isomorphism theorem. **10**

b) Prove that the set Aut (G) of all automorphisms of a group G is a group under compositions of mappings, and $I_n(G) \Delta \text{ Aut}(G)$ moreover $\frac{G}{Z(G)} \simeq In(G)$. **10**

OR

c) Let G be a group and X be a set then prove that **10**
i) If X is a G-set, then action of G on X induces a homomorphism $\phi = G \to Sx$.
ii) Any homomorphism $\phi = G \to Sx$ induces an action of G onto X.

d) Show that every group is isomorphic to a permutation group. **10**

UNIT – II

2. a) Show that a group G is solvable iff G has a normal series with abelian factors. Further prove that a finite group is solvable iff its composition factors are cyclic groups of prime order. **10**

b) Let G be a nilpotent group. Then prove that every subgroup of G and every homomorphic image of G are nilpotent. **10**

OR

c) An is a normal sub group of S_n. If n>1, An is of index 2 in S_n and hence, prove that $|A_n| = \frac{1}{2} n!$ **10**

d) Let G be a group show that if G is solvable then every subgroup of G and every homomorphic image of G are solvable. Show also that, if N is a normal subgroup of G such that N and G/N are solvable, then G is solvable. **10**

UNIT – III

3. a) Prove that there are no simple group of order 63, 56 and 36. **10**

b) Let A be a finite abelian group and P be a prime. If P divides |A|, then show that A has an element of order P. **10**

OR

c) Let G be a finite group and Let P be a prime. Then prove that all sylow P-subgroups of G are conjugates and their number n_p divides 0 (G) and satisfies. **10**

$n_p \equiv 1 \pmod{p}$

d) If G is a group of order pq where p and q are prime number such that p>q and q x (p-1) then show that G is cyclic. **10**

UNIT - IV

4. a) Let f be the homomorphism of a ring R into a ring S with Kernel N. Then prove that $R/N \simeq \text{In} f$. **10**

b) If a ring R has unity, then prove that every ideal I in the matrix ring R_n is of the form A_n, where A is some ideal in R. **10**

OR

c) Let $(Ni)_{i \in \wedge}$ be a family of R-submodules of an R-module. Then prove that $\bigcap_{i \in \wedge} Ni$ is also an R-submodule. **10**

d) Show that The submodules of the quotient module M/N are of the form U/N where U is a submodule of M containing N. **10**

5. a) Show that Every subgroup of a cyclic group is cyclic. **5**

b) Prove that every finite group has a composition series. **5**

c) If a group of order p^n contains exactly one subgroup each of orders $p, p^2, \ldots\ldots. p^{n-1}$ then prove that it is cyclic. **5**

d) Let R be a commutative ring with unity in which each ideal is prime then prove that R is a field. **5**

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