M.Sc.- I (Mathematics) (C.B.C.S. Pattern) Sem-II

PSCMTHT10-Paper-IV - Differential Geometry

P. Pages : 2 **GUG/W/19/11215**

Time : Three Hours * 1 6 0 3 * Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that the metric is invariant under a parametric transformation. **10**

b) Find the coefficient of the direction which makes an angle $\frac{\pi}{2}$ with the direction whose coefficient are (ℓ, m) **10**

OR

c) Prove that when $v = c$ for all value s of u, a necessary and sufficient condition that the curve $v = c$ is a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$. **10**

d) Show that the curve bisecting the angle between the parametric curves are given by $Edu^2 - G\,dv^2 = 0$. **10**

UNIT – II

2. a) Find the Gaussian curvature at a point (u, v) of the anchor ring **10**
$r = ((b + a\cos u)\cdot\cos v, (b + a\cos u)\cdot\sin v,\ a\sin u)\ \ 0 < u,\ v < 2\pi$

b) State and prove Gauss-Bonnet theorem. **10**

OR

c) Prove that in the geodesic polar form, the Gaussian curvature $k = \frac{-g_{11}}{g}$ where $g = \sqrt{G}$ of the surface. **10**

d) Prove that every helix on a cylinder is a geodesic and conversely every geodesic on a cylinder is a helix. **10**

UNIT – III

3. a) If K is a normal curvature in direction making an angle ψ with the principal direction v = constant then prove that $K = K_a \cdot \cos^2\psi + K_b \cdot \sin^2\psi$. **10**

b) Prove that the principal curvatures are given by roots of the equation **10**
$K^2(EG-F^2)-K(EN+GL-2FM)+LN-M^2=0$.

OR

c) Find the umbilics of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **10**

d) Prove that if the surface is a developable, then its Gaussian curvature shall be zero. **10**

UNIT – IV

4. a) Prove that the only compact surfaces of class ≥ 2 for which every point is an umbilic are spheres. **10**

b) State and prove Hilbert's lemma. **10**

OR

c) Show that, if on a compact surface S the curvature every where exceeds $1/a^2$ then the maximum distance between any two points cannot exceed πa. **10**

d) Prove that in order that the geodesic distance AB should be the shortest distance, it is necessary and sufficient that B lies between A and its conjugate point A_1. **10**

5. a) If w is the angle between the parametric curves at the point of intersection then prove that $\tan w=\frac{H}{F}$ **5**

b) If (λ, μ) is the geodesic curvature vector of a curve, then Prove that **5**

$$Kg=\frac{-H\lambda}{Fu'+Gv'}=\frac{H\mu}{Hu'+Fv'}$$

c) Show that the meridians and parallels of a surface of revolution are its line of curvature. **5**

d) State the characterizations of complete surface. **5**

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