M.Sc. (Part-I) (Mathematics) (C.B.C.S. Pattern) Sem-II

PSCMTHT09-Paper-V - Classical Mechanics

P. Pages : 2 **GUG/W/19/11214**

Time : Three Hours * 1 6 0 4 * Max. Marks : 100

Notes : 1. Solve all five questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that the shortest distance between two points in a plane is a straight line. **10**

b) By the minimum surface of revolution obtain the equation of catenary **10**

$$x = a \cosh\left(\frac{y-b}{a}\right)$$

OR

c) Show that the hoop rolls down the inclined plane with only one half the acceleration it would have slipping down a frictionless plane and the frictional force of constraints is **10**

$$\lambda = \frac{M.g \sin\phi}{2}$$

d) Discuss the brachistochrone problem to find the curve joining two points. **10**

UNIT – II

2. a) Deduce the Hamilton's equation of motion of a particle of mass m in cartesian co-ordinates. **10**

b) State and prove the principle of least action. **10**

OR

c) Obtain the Hamilton's canonical equations, **10**

$$\frac{\partial H}{\partial P_i} = \dot{q}_i, \quad \frac{\partial H}{\partial q_i} = -\dot{p}_i$$

$$\frac{\partial H}{\partial t} = \frac{-\partial L}{\partial t}$$

d) The kinetic and potential energies of a particle are respectively given by **10**

$$T = \frac{1}{2} m \dot{r}^2, \quad V = \frac{1}{r}\left(1 + \frac{r^2}{c^2}\right)$$

Find the Hamiltonian of the system and show that the system is not conservative.

UNIT – III

3. a) Show that fundamental Poisson brackets are invariant under canonical transformation. **10**

b) Explain the symplectic approach to canonical transformations and obtain the necessary condition $MJ\tilde{M} = J$. **10**

OR

c) If $F_1(q, Q, t)$ and $F = F_2(q, p, t)$ are generating functions of canonical transformation then prove that **10**

i) $K = H + \frac{\partial f_1}{\partial t}$ & ii) $K = H + \frac{\partial f_2}{\partial t}$

d) Show that the value of the Poisson bracket [Q, P] implies the symplectic condition **10**
i.e. $\frac{\partial Q}{\partial p} \cdot \frac{\partial \phi}{\partial Q} = -\frac{\partial Q}{\partial p} \cdot \frac{\partial \Psi}{\partial q}$

UNIT – IV

4. a) State and prove the Liouville's theorem. **10**

b) In a symmetry groups of mechanical systems obtain the identities. **10**

$\left[L_i, L_j\right] = \in_{ijk} L_k$

$\left[D_i, L_j\right] = \in_{ijk} D_k$

$\left[D_i, D_j\right] = \in_{ijk} L_k$

OR

c) Show that the constants of the motion are the generating functions of those infinitesimal canonical transformations that leave the Hamiltonian invariant. **10**

d) Discuss the symmetry groups of mechanical system. **10**

5. a) Show that generalized momentum conjugate to a cyclic co-ordinate is conserved. **5**

b) Obtain the Jacobi's form of least action principle. **5**

c) Show that directly that the transformation $Q = \log\left(\frac{1}{q}\sin P\right)$, $p = q \cot P$ is canonical. **5**

d) Obtain the relations **5**

$$\dot{q}_i = [q_i, H], \dot{p}_i = [p_i, H]$$

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